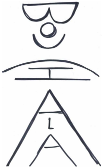
K čemu je dobrý sinus?

Jiří Bouchala



Katedra
aplikované
matematiky

28. ledna 2019, přednáška v semináři Škomam


Bouchala
2018

$$\forall x \in \mathbb{R} : \ \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots$$

$$\forall x \in \mathbb{R}: \ \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \cdots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \ldots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

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$\sin(22)$

$$\forall x \in \mathbb{R}: \ \sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots + (-1)^n \frac{x^{2n+1}}{(2n+1)!} + \dots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}.$$

$$\sin(22) \doteq \sum_{n=0}^{50} (-1)^n \frac{22^{2n+1}}{(2n+1)!}.$$

$$\sum_{n=0}^{50}(-1)^n\frac{x^{2n+1}}{(2n+1)!}=$$

$$x-\frac{1}{6}x^3+\frac{1}{120}x^5-\frac{1}{5040}x^7+\frac{1}{362880}x^9-\frac{1}{39916800}x^{11}+\frac{1}{6227020800}x^{13}-\frac{1}{1307674368000}x^{15}+\frac{1}{355687428096000}x^{17}-$$
$$\frac{1}{121645100408832000}x^{19}+\frac{1}{51090942171709440000}x^{21}-\frac{1}{25852016738884976640000}x^{23}+\frac{1}{15511210043330985984000000}x^{25}-$$
$$\frac{1}{10888869450418352160768000000}x^{27}+\frac{1}{8841761993739701954543616000000}x^{29}-\frac{1}{8222838654177922817725562880000000}x^{31}+$$
$$\frac{1}{8683317618811886495518194401280000000}x^{33}-\frac{1}{10333147966386144929666651337523200000000}x^{35}+$$
$$\frac{1}{13763753091226345046315979581580902400000000}x^{37}-\frac{1}{20397882081197443358640281739902897356800000000}x^{39}+$$
$$\frac{1}{33452526613163807108170062053440751665152000000000}x^{41}-\frac{1}{60415263063373835637355132068513997507264512000000000}x^{43}+$$
$$\frac{1}{119622220865480194561963161495657715064383733760000000000}x^{45}-$$
$$\frac{1}{258623241511168180642964355153611979969197632389120000000000}x^{47}+$$
$$\frac{1}{608281864034267560872252163321295376887552831379210240000000000}x^{49}$$

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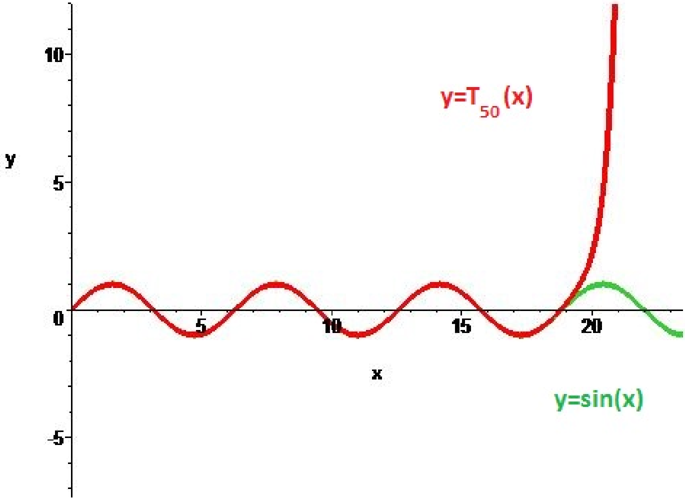
10
5
0
-5
y
5
10
15
20
x
y=T50(x)
y=sin(x)

$$\sin x = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

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$$\sin(22) - (-0,00887) \doteq \color{red}{0,0000187}$$


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2018

... lze dokázat, že pro každou funkci $f \in C([0, 2\pi])$ platí

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \big(a_n \cos(nx) + b_n \sin(nx)\big),$$

kde

$$a_n = \frac{1}{\pi} \int_0^{2\pi} f(x) \cos(nx) \, \mathrm{d}x$$

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Takže

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Takže

$$f(x) \doteq \frac{a_0}{2} + \sum_{n=1}^{N} \left(a_n \cos(nx) + b_n \sin(nx)\right) =: f_N(x).$$

Příklad.

$$f(x) := \frac{1}{150}x^2(x-2\pi)^3(x-1)^2\sin(3x), \quad f_4(x) := \ldots, \quad f_5(x) := \ldots$$

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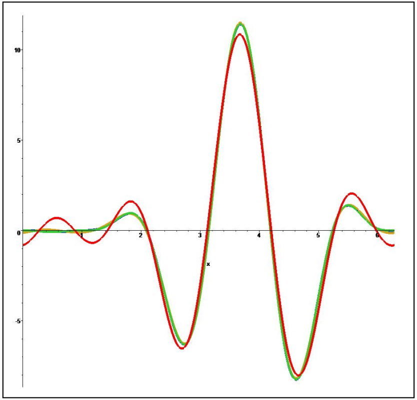

$$f(x) := \frac{1}{150}x^2(x-2\pi)^3(x-1)^2\sin(3x) \doteq \frac{a_0}{2} + \sum_{n=1}^{6}\left(a_n\cos(nx) + b_n\sin(nx)\right).$$

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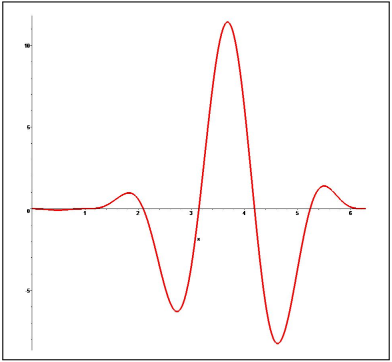

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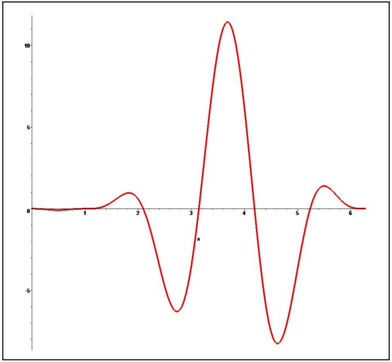

$$f \approx [-0,1; -0,71; -0,1; 1,74; 2,38; 0; -4,43; -1,74; 2,38; 0,68; -0,11; 0,1; -0,03]$$


Bouchala
2017

Děkuji vám za pozornost!