Chyby při matematickém modelování aneb co se nepovedlo

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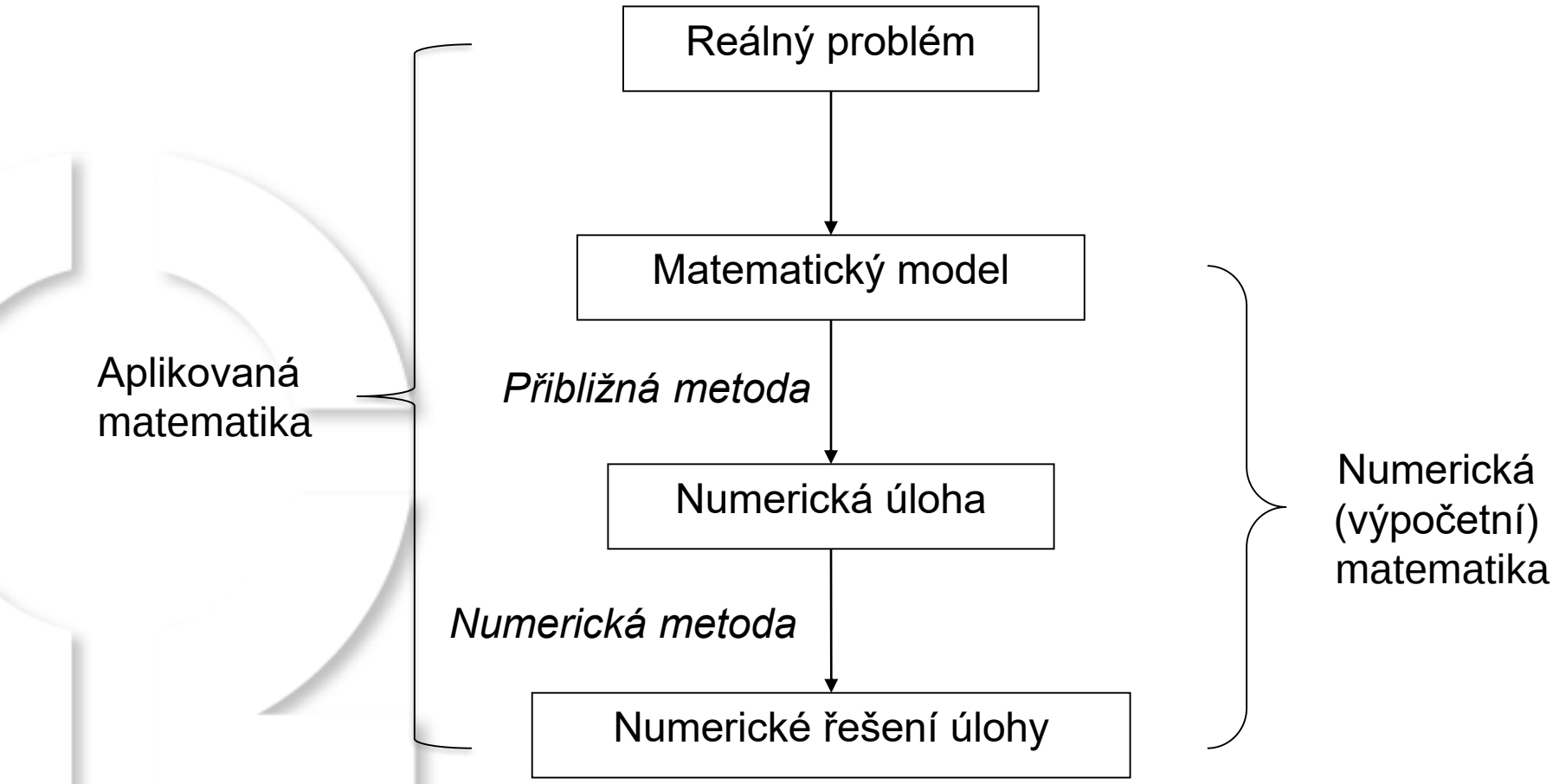
Obsah

- **Co to je matematické modelování a k čemu je dobré?**
- **Co se může pokazit?**
- **A co se pokazilo?**

Matematické modelování



Matematické modelování – příklad 1

Deformace struny

Lineární model – malé deformace

Rovnice struny

Diskretizace geometrie úlohy – metoda sítí

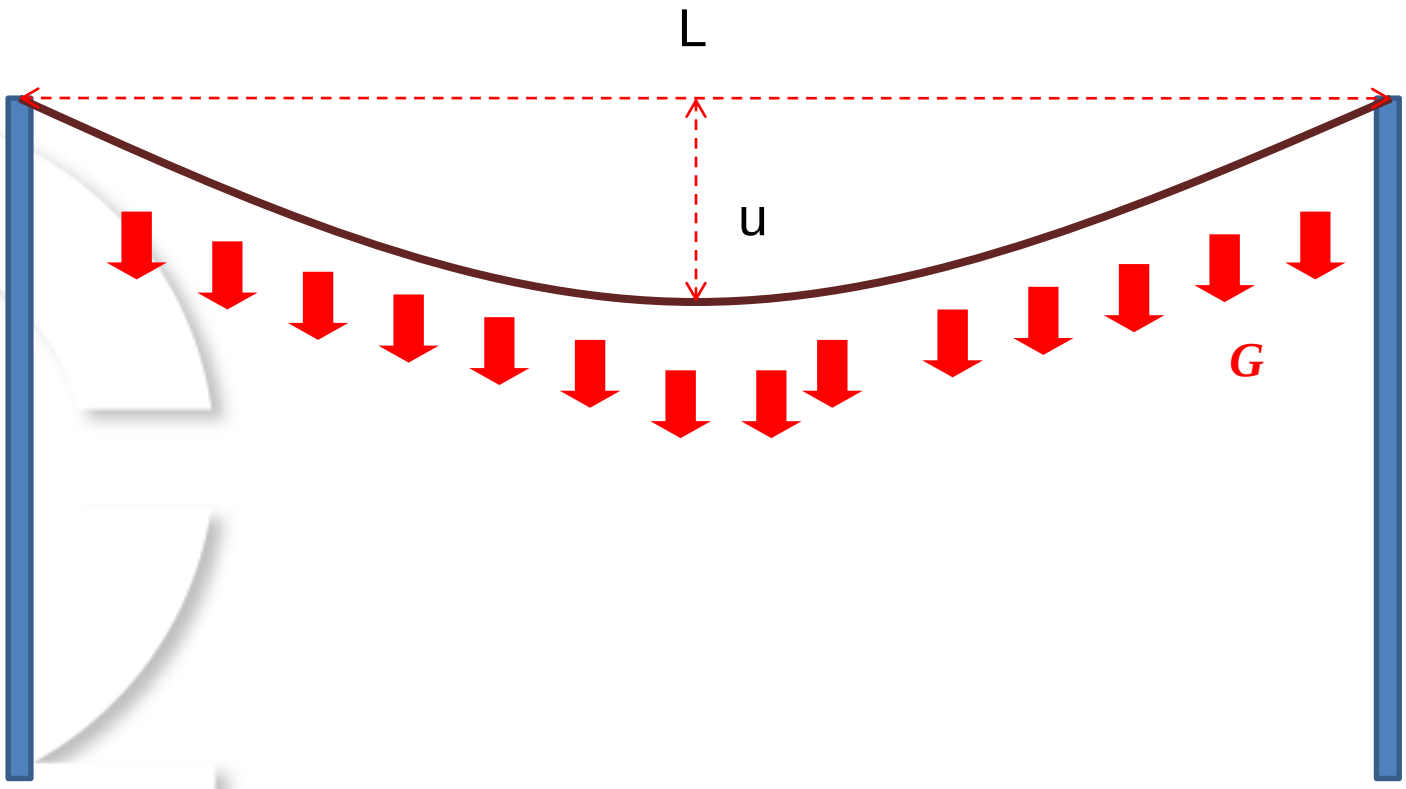
Diskretizovaná rovnice struny

Řešení lineární soustavy rovnic – Gaussova eliminace nebo jiná iterační metoda

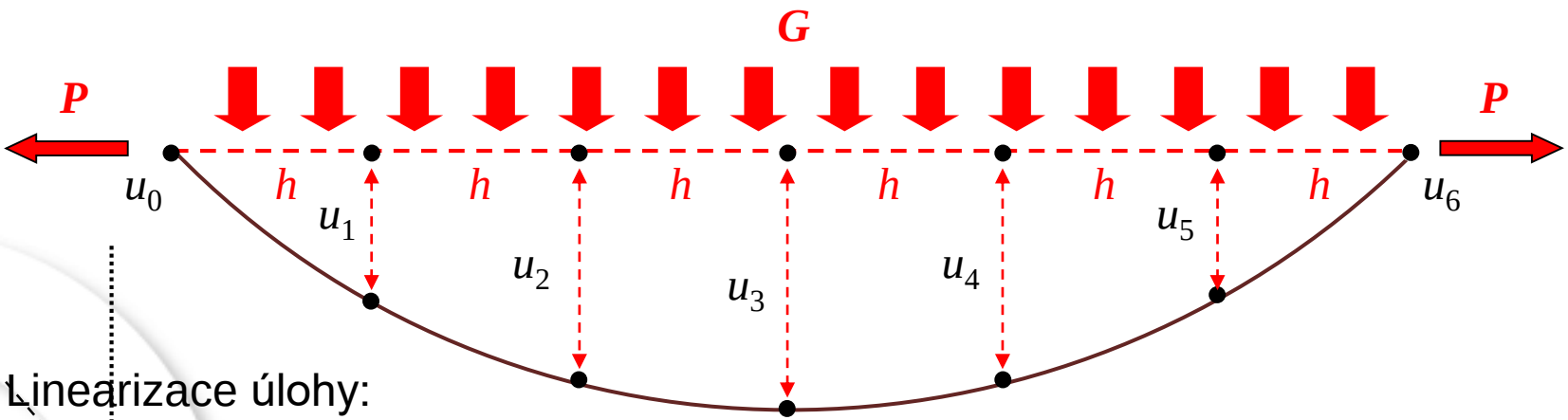
Odhad deformace struny

Numerická (výpočetní) matematika

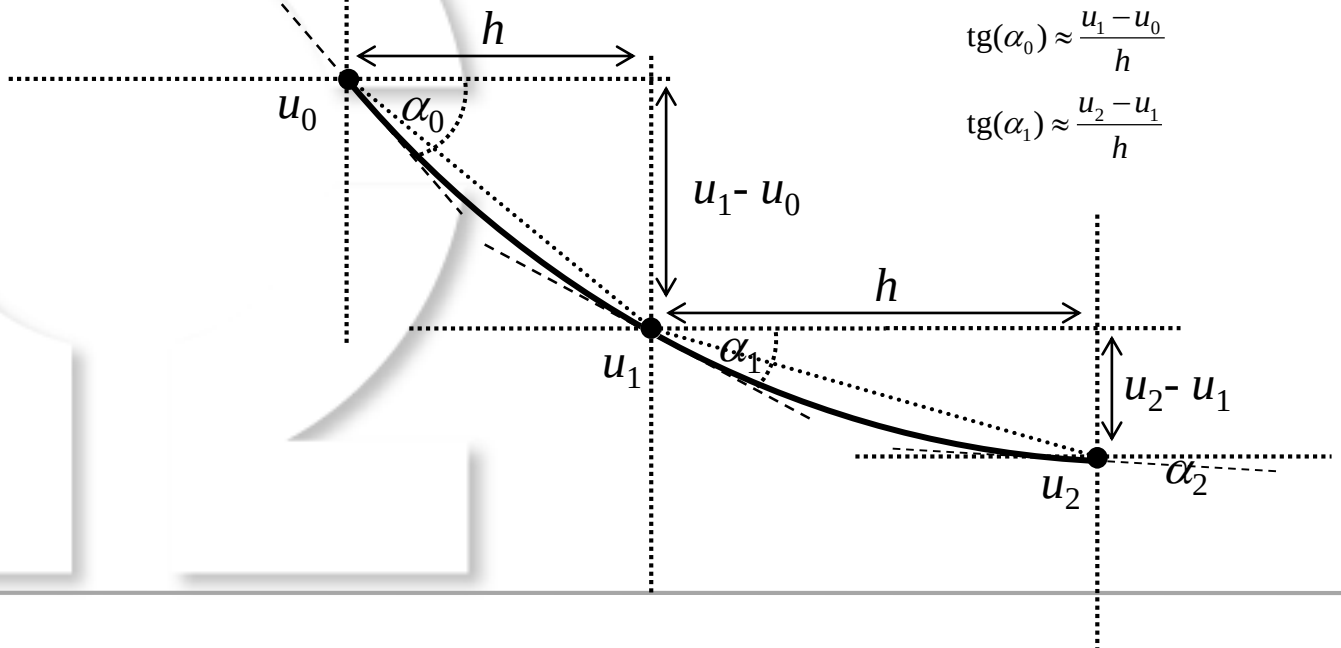
Příklad 1 - Deformace struny



Příklad 1 - Diskretizace struny



Linearizace úlohy:



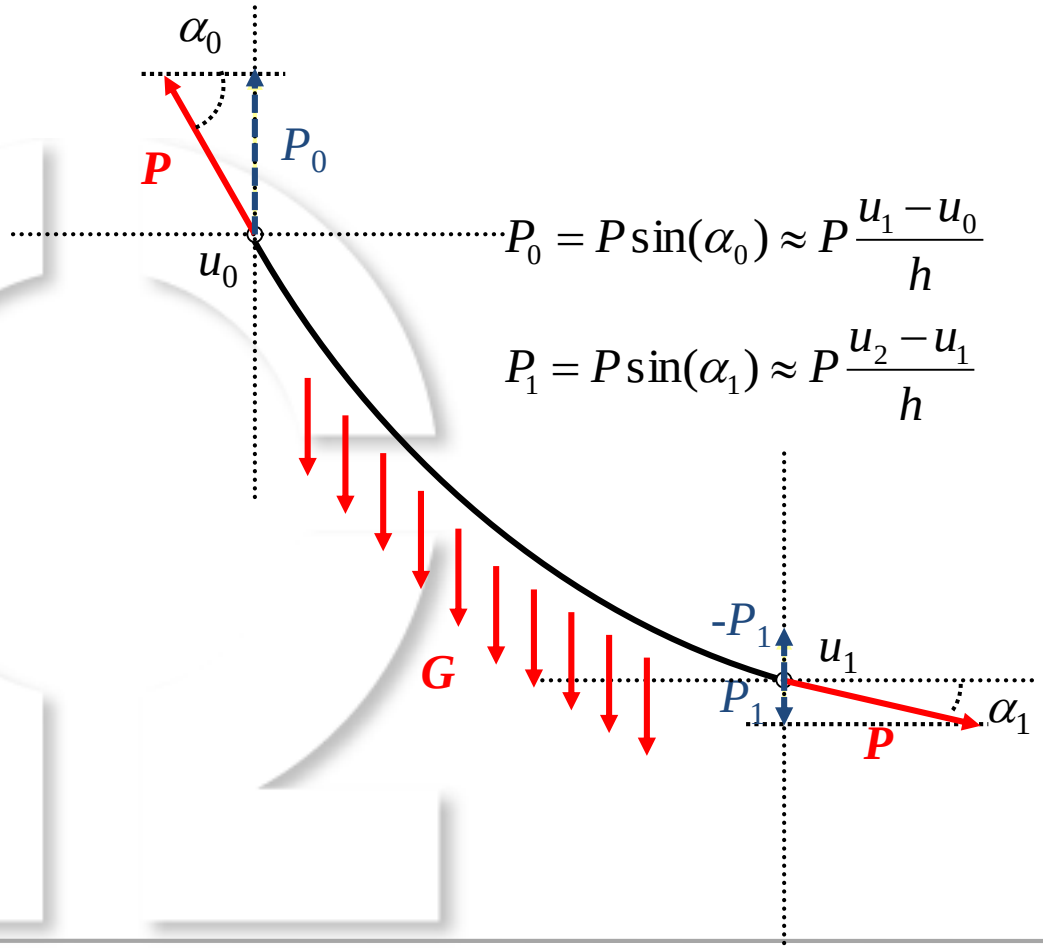
$$\mathrm{tg}(\alpha_0) \approx \frac{u_1 - u_0}{h}$$

$$\mathrm{tg}(\alpha_1) \approx \frac{u_2 - u_1}{h}$$

Malé deformace:

$$\mathrm{tg}(\alpha) \approx \sin(\alpha)$$

Příklad 1 - Diskretizace struny



$$P_0 = P\sin(\alpha_0) \approx P\frac{u_1 - u_0}{h}$$

$$P_1 = P\sin(\alpha_1) \approx P\frac{u_2 - u_1}{h}$$

$$P_0 + (-P_1) = hG$$

$$P\frac{u_1 - u_0}{h} - P\frac{u_2 - u_1}{h} = hG$$

$$-u_0 + 2u_1 - u_2 = \frac{h^2 G}{P}$$

Příklad 1 – Odhad deformace struny

Soustava lineárních rovnic:

$$
\begin{aligned}
-u_0 + 2u_1 - u_2 &= h^2F \\
-u_1 + 2u_2 - u_3 &= h^2F \\
-u_2 + 2u_3 - u_4 &= h^2F \\
-u_3 + 2u_4 - u_5 &= h^2F \\
-u_4 + 2u_5 - u_6 &= h^2F
\end{aligned}
$$

$$F = \frac{G}{P} = \frac{\rho_l g}{\text{"tuhost"}}$$

$$F = \frac{0.85 \cdot 9.81}{300} = 0.0278$$

$$L = 12m \Rightarrow h = \frac{L}{6} = 2m$$

$$u_0 = u_6 = 0$$

$$
\begin{aligned}
2u_1 - u_2 &= 0.1112 \\
-u_1 + 2u_2 - u_3 &= 0.1112 \\
-u_2 + 2u_3 - u_4 &= 0.1112 \\
-u_3 + 2u_4 - u_5 &= 0.1112 \\
-u_4 + 2u_5 &= 0.1112
\end{aligned}
$$

Au=b

$$
\begin{aligned}
u_1 &= 0.2780 \\
u_2 &= 0.4448 \\
u_3 &= 0.5004 \\
u_4 &= 0.4448 \\
u_5 &= 0.2780
\end{aligned}
$$

Matematické modelování – příklad 2

Tvarová optimalizace pro kontaktní úlohu se třením

Variační nerovnost – zanedbání tření

Optimalizační úloha

Diskretizace geometrie úlohy – metoda konečných prvků

Diskretizovaná optimalizační úloha

Metoda pro optimalizaci spojitě diferencovatelné funkce

Odhad optimálního tvaru

Numerická (výpočetní) matematika

Shape optimization

- Shape optimization in contact problem with Coulomb friction

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$$\begin{aligned} &\min \Theta(\boldsymbol{\alpha}) \\ &\text{subject to} \\ &\qquad \boldsymbol{\alpha} \in U_{ad}, \end{aligned}$$

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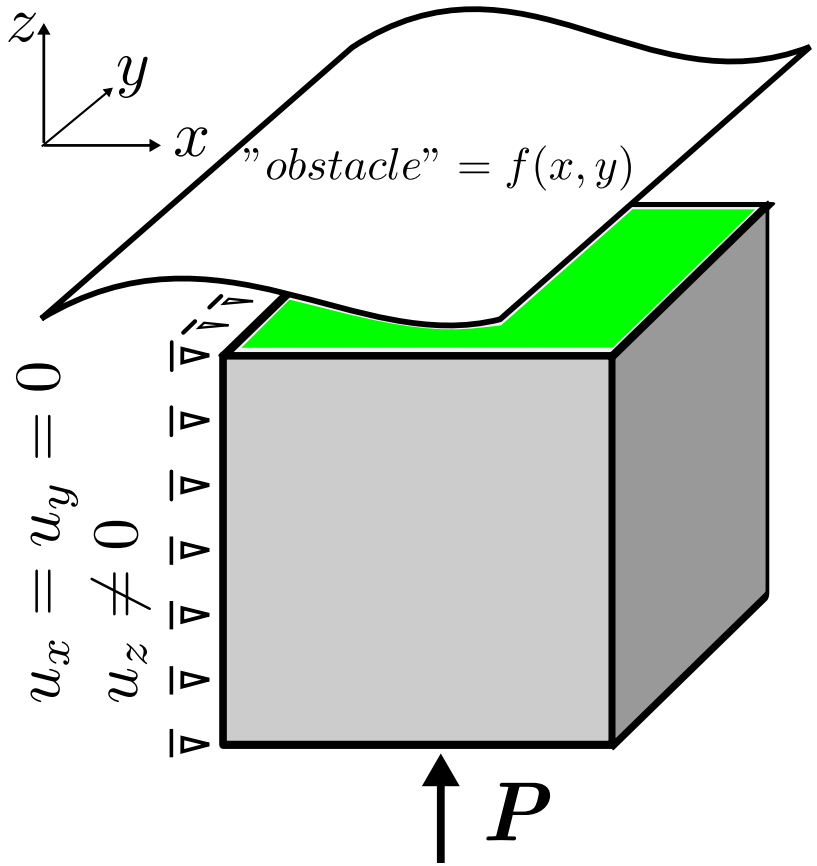
where $\Theta(\boldsymbol{\alpha}) := \mathcal{J}(\boldsymbol{\alpha}, \mathcal{S}(\boldsymbol{\alpha}))$

Shape optimization

Signorini problem - the meaning of control variables α



Shape optimization - Indirect approach (Neglecting friction)

- Shape optimization problem is solved with $\mathcal{F} = 0$.

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Shape optimization - Indirect approach (Neglecting friction)

- Shape optimization problem is solved with $\mathcal{F} = 0$.
- Function $\Theta(\boldsymbol{\alpha}) := \mathcal{J}(\boldsymbol{\alpha}, \mathcal{S}(\boldsymbol{\alpha}))$ is differentiable.
- Does the optimized shape computed with $\mathcal{F} = 0$ approximate the optimized shape of original problem?

Numerical experiments: Signorini problem II (without friction)

$$\min_{\boldsymbol{\alpha}} \|\boldsymbol{\lambda}_c(\boldsymbol{\alpha})\|_4^4$$
subject to
$$\boldsymbol{\alpha} \in U_{ad}$$

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number of subdomains 8

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number of subdomains 8

number of design variables 36

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nodal degrees of freedom 8 232

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elastic body:
Young modulus $E := 1.1 \cdot 10^5$ MPa, Poisson's ratio $\nu := 0.33$

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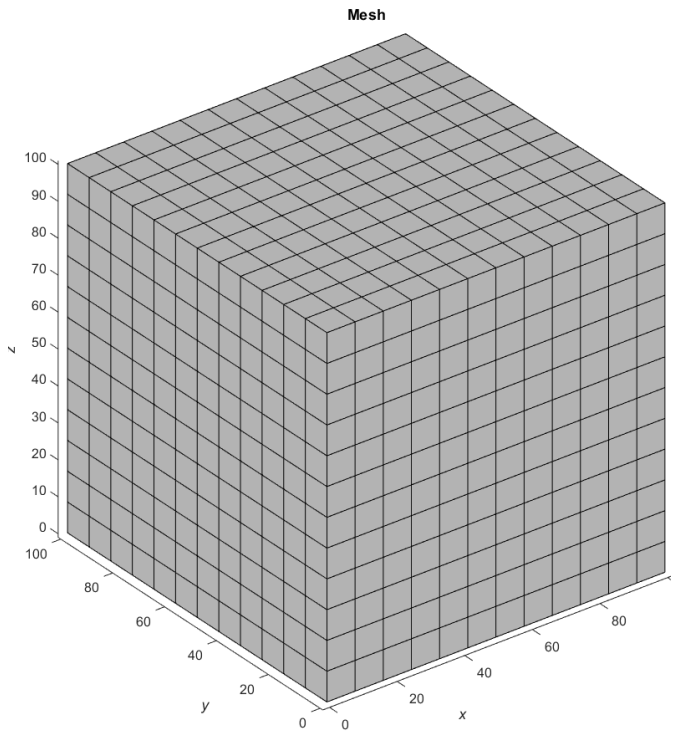
Young modulus $E := 1.1 \cdot 10^5$ MPa, Poisson's ratio $\nu := 0.33$

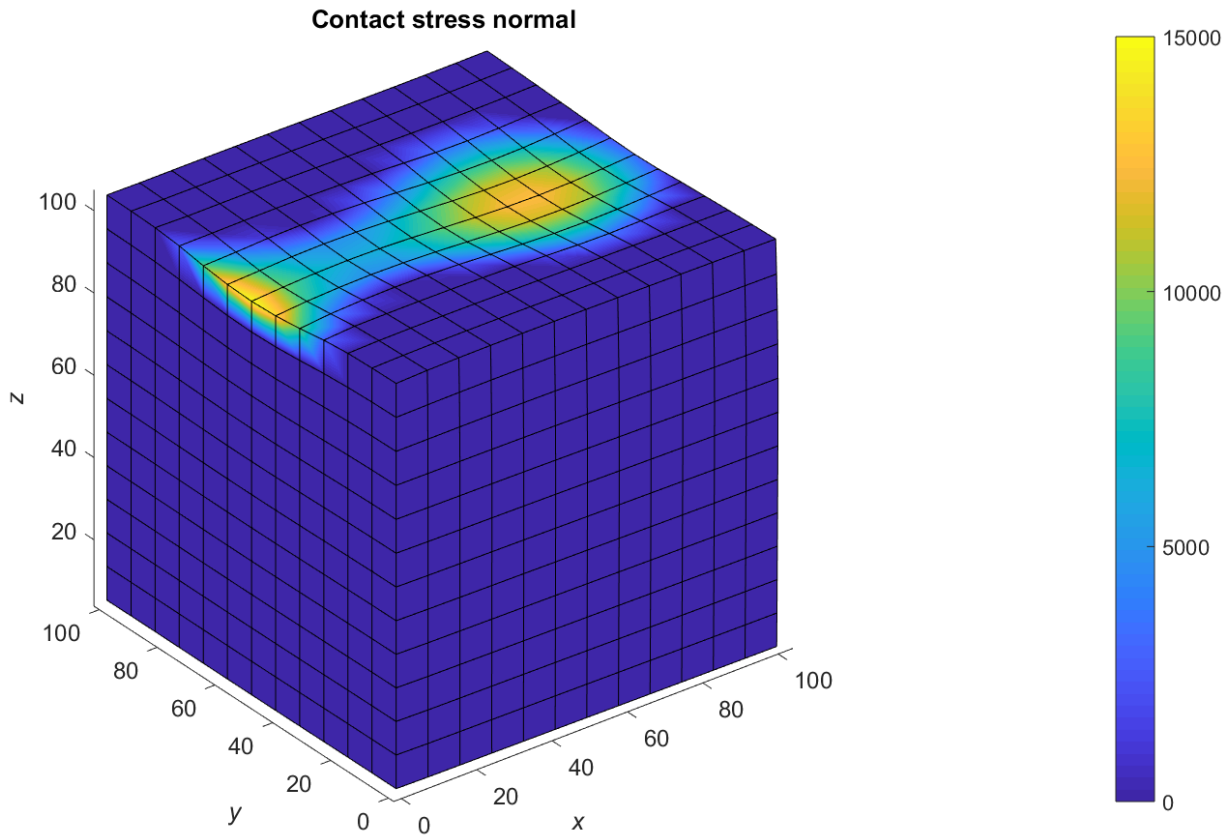
coefficient of Coulomb friction $\mathcal{F} := 0$

body traction $P := 3000 \ \frac{\mathrm{N}}{\mathrm{mm}^2}$

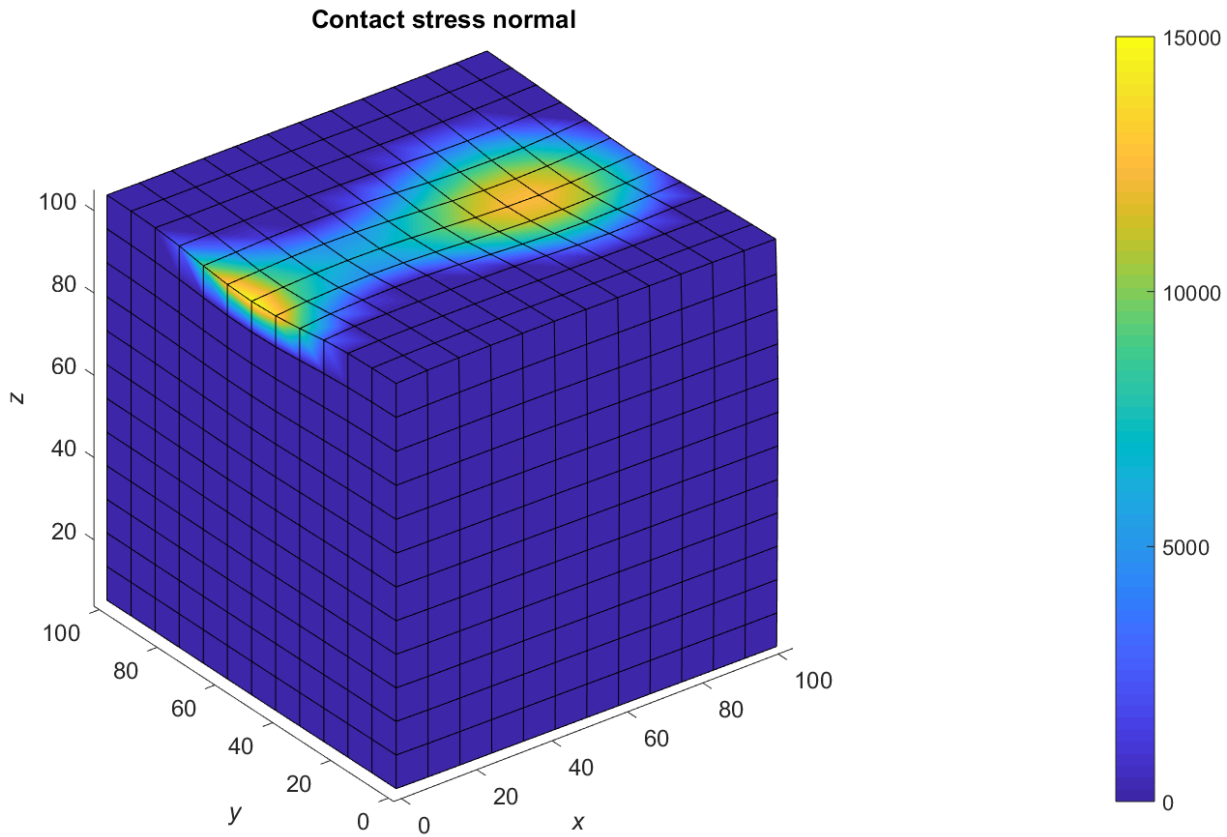
Signorini problem II (without friction): Initial design



Signorini problem II (without friction): Normal contact stress for initial design

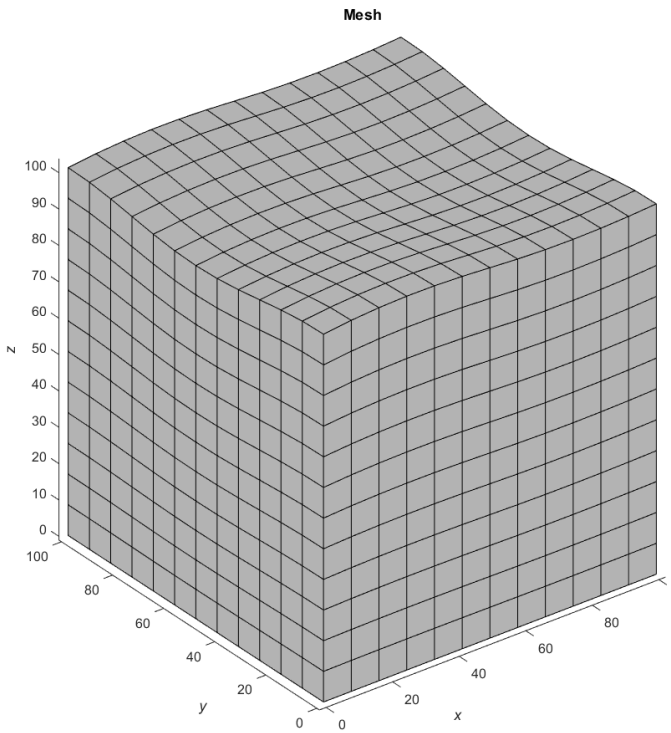


Signorini problem II (without friction): Normal contact stress for initial design

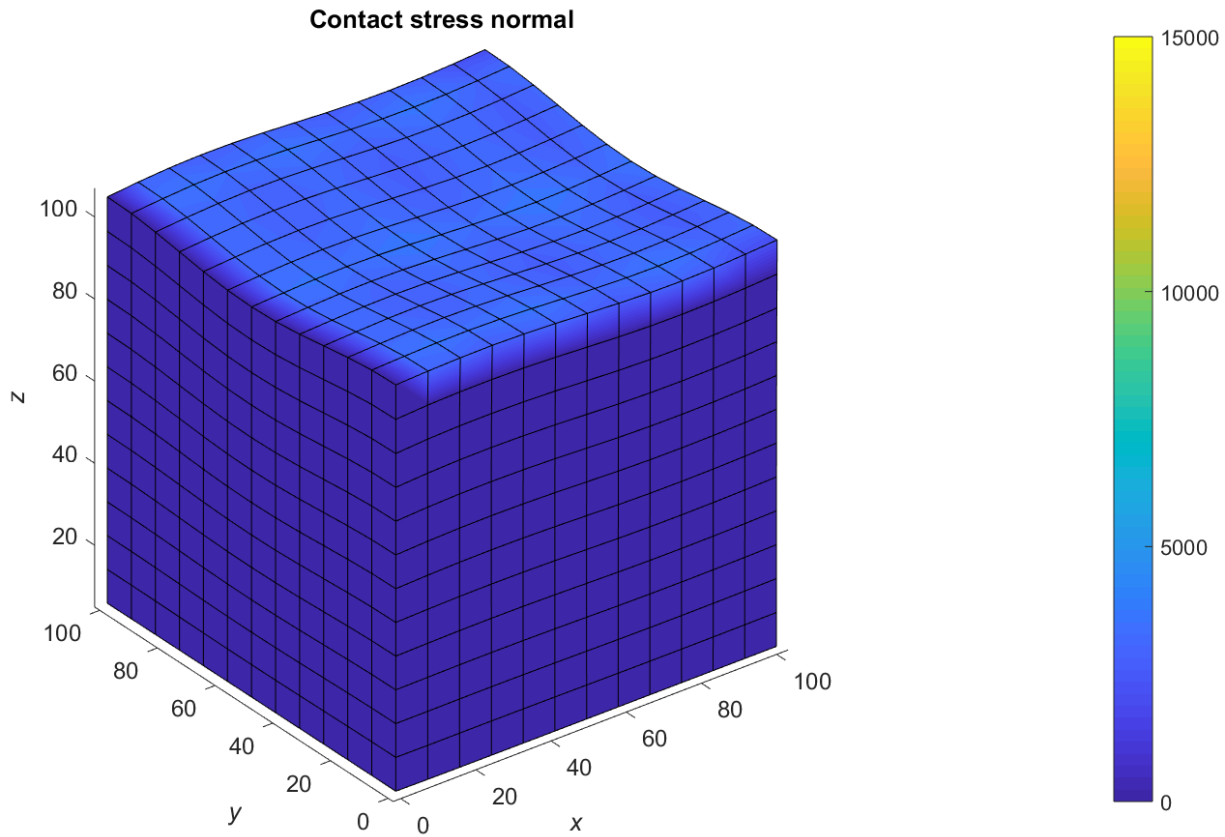


$$\|\boldsymbol{\lambda}_c(\boldsymbol{\alpha})\|_4^4 = 4.2334 \cdot 10^{17}$$

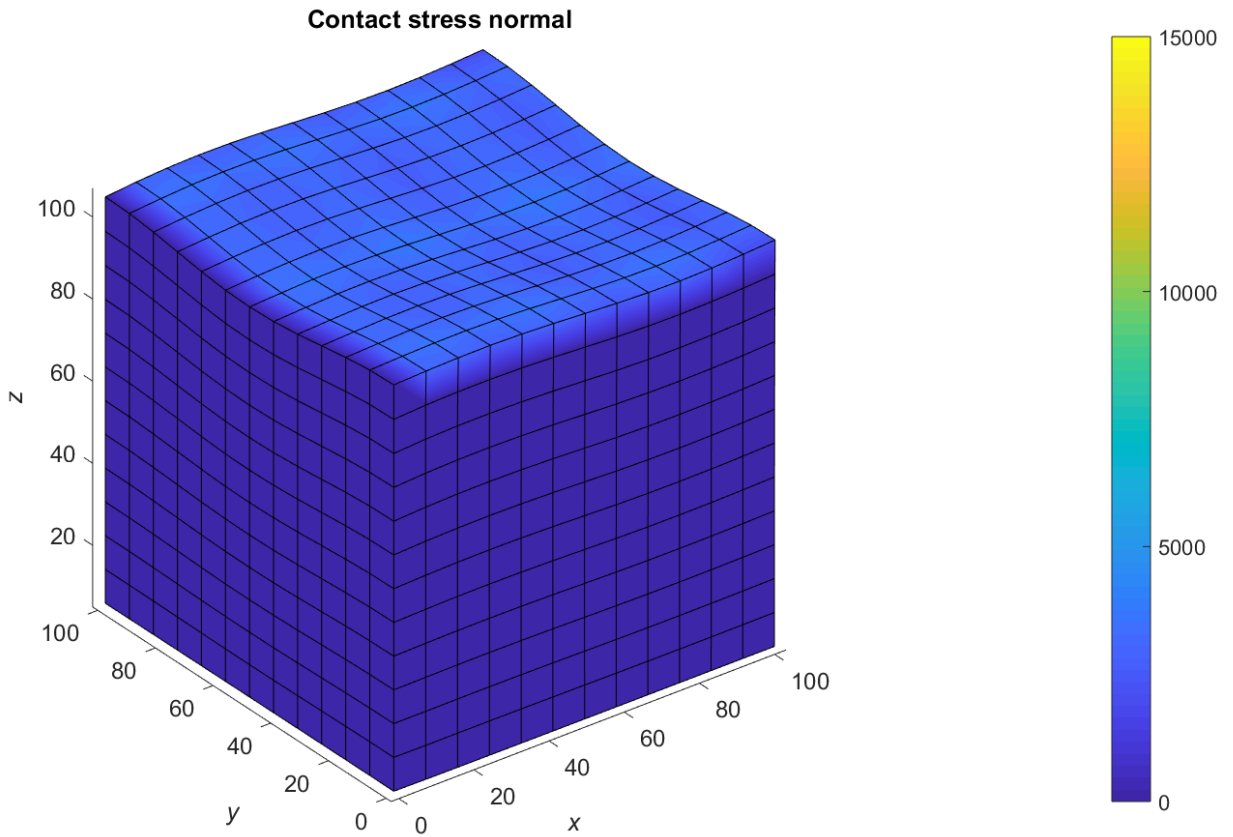
Signorini problem II (without friction): Optimized design



Signorini problem II (without friction): Normal contact stress for optimized design

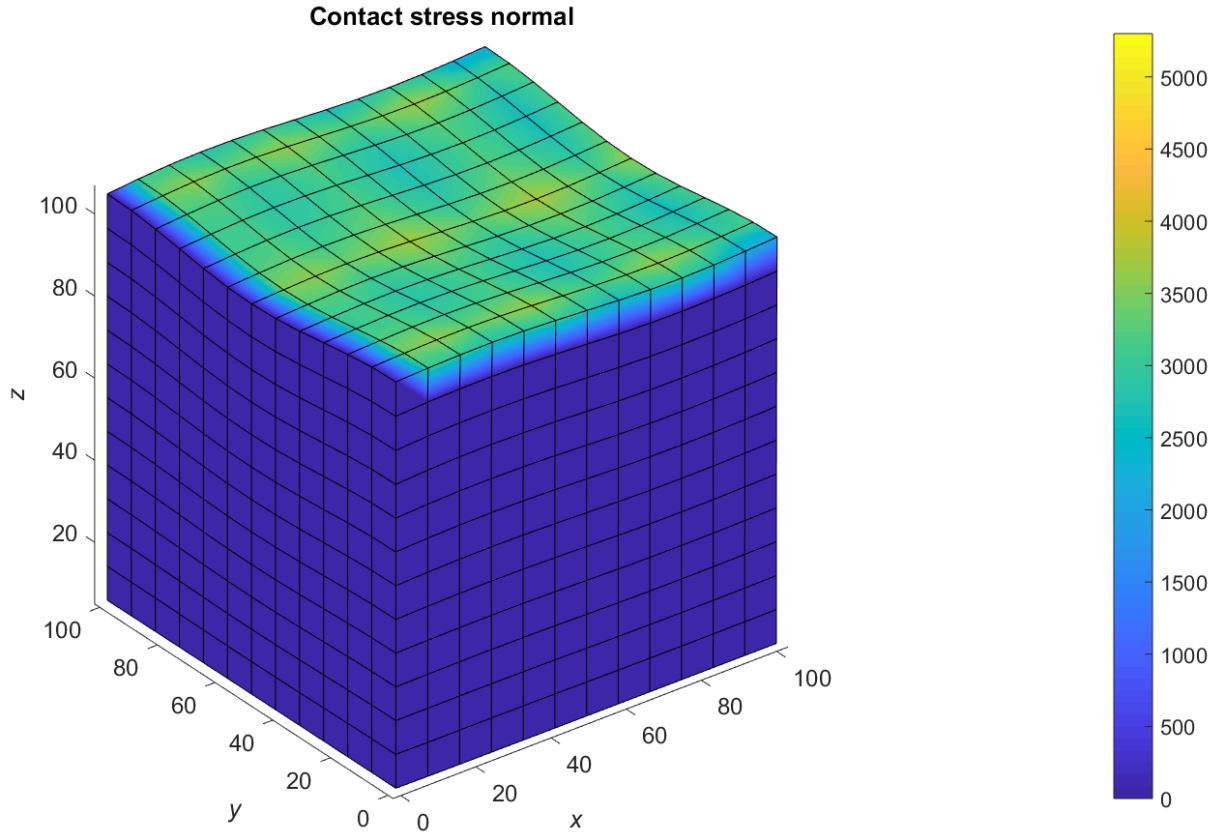


Signorini problem II (without friction): Normal contact stress for optimized design

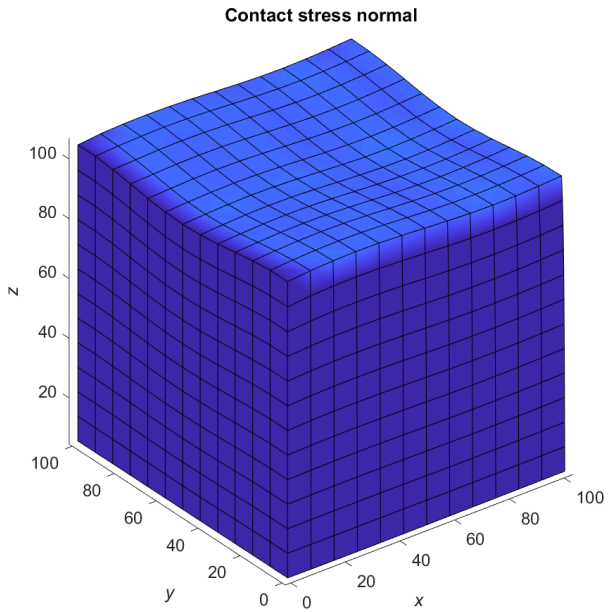


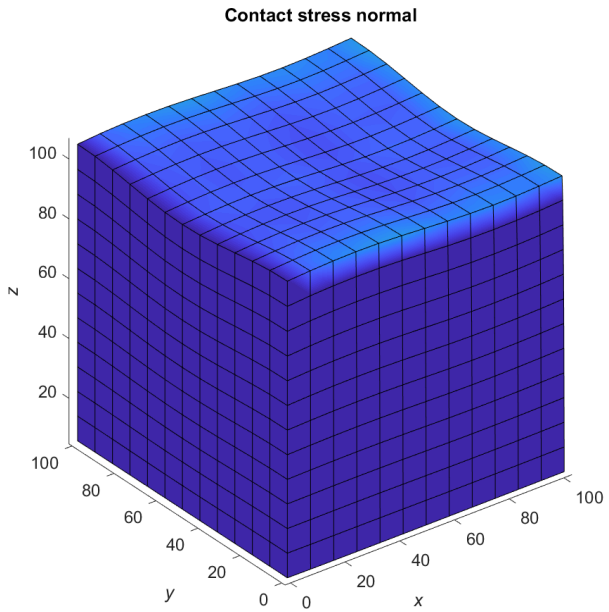
$$\|\boldsymbol{\lambda}_c(\boldsymbol{\alpha})\|_4^4 = 1.5207 \cdot 10^{16}$$

Signorini problem II (without friction): Normal contact stress for optimized design (smaller normal contact stress range scale)

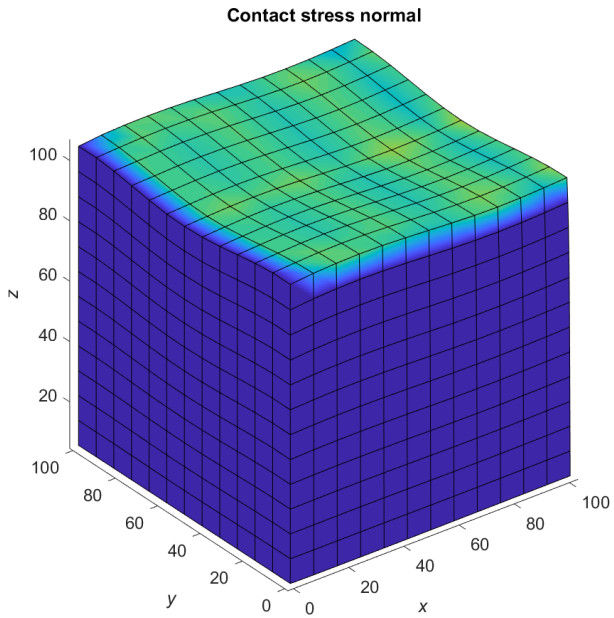


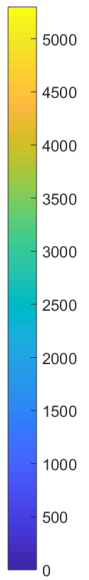
Signorini problem I and II - comparison 1 (both with friction): Normal contact stress for optimized design

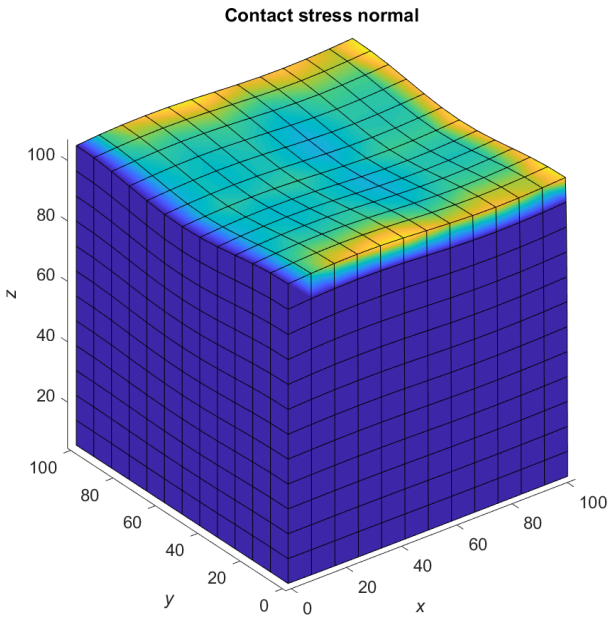


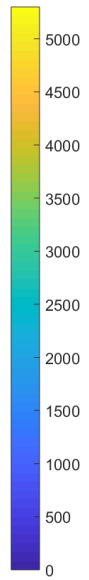


Signorini problem I and II - comparison 2 (both with friction): Normal contact stress for optimized design

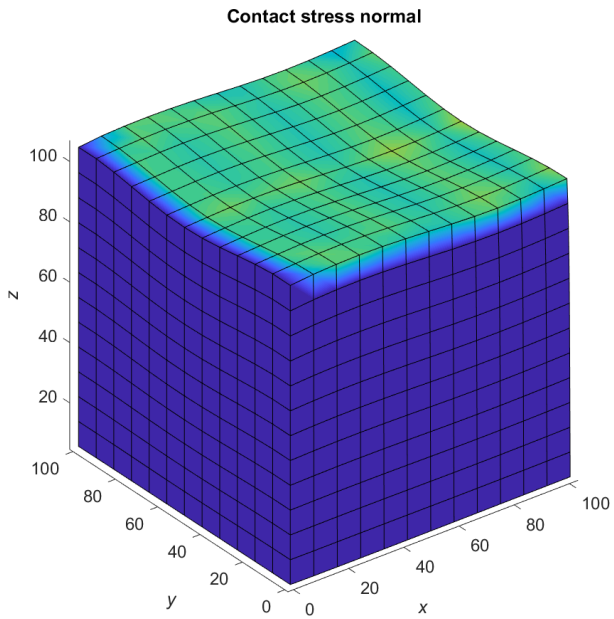


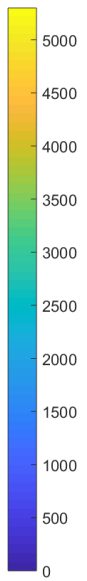




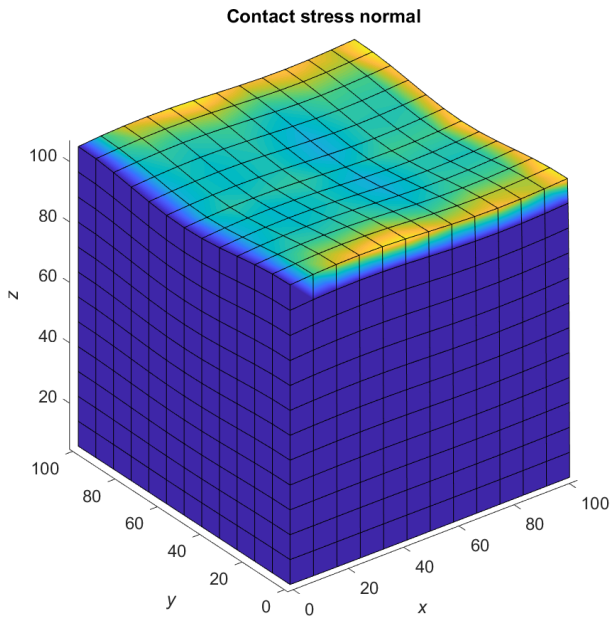


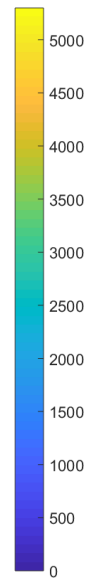
Signorini problem I and II - comparison 2 (both with friction): Normal contact stress for optimized design





$$\|\boldsymbol{\lambda}_c(\boldsymbol{\alpha})\|_4^4 = 1.4193 \cdot 10^{16}$$





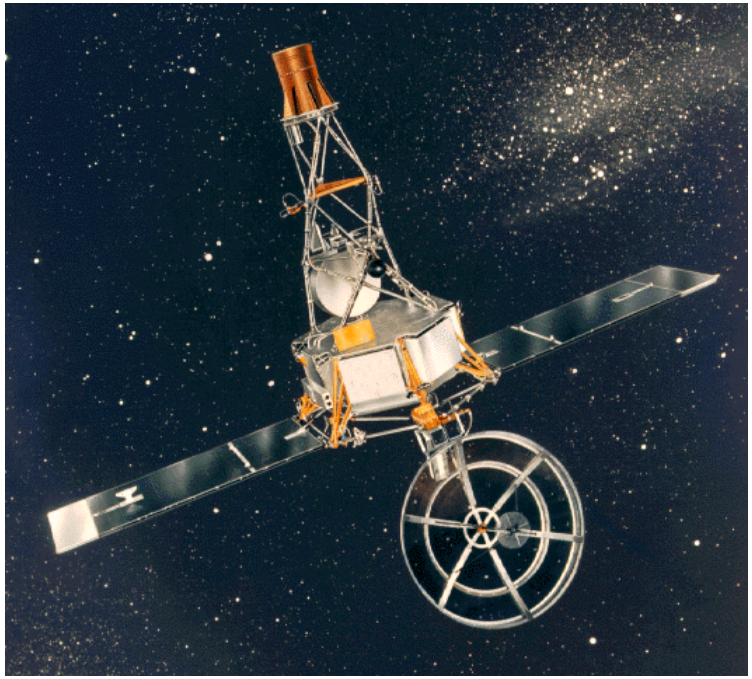
$$\|\boldsymbol{\lambda}_c(\boldsymbol{\alpha})\|_4^4 = 2.4217 \cdot 10^{16}$$

Chyby, které se mohou objevit

- **Chyba matematického modelu**
- **Chyba metody, chyba aproximace**
- **Chyby v kódu algoritmu**
- **Chyby ve vstupních datech**
- **Zaokrouhlovací chyby, chyby zápisu čísla v počítači**

Mariner 1

- Zničen 22. 7. 1962 při letu k Venuši **kvůli chybě v kódu algoritmu**
- Celková škoda cca 550 miliónů USD



Systém Patriot

- Selhal 25. 2. 1991 během první války v Perském zálivu **kvůli zaokrouhlovací chybě**
- Výsledkem byl dopad irácké rakety Scud na americká kasárna a smrt 28 amerických vojáků a zranění cca 100 dalších osob

Sleipner A

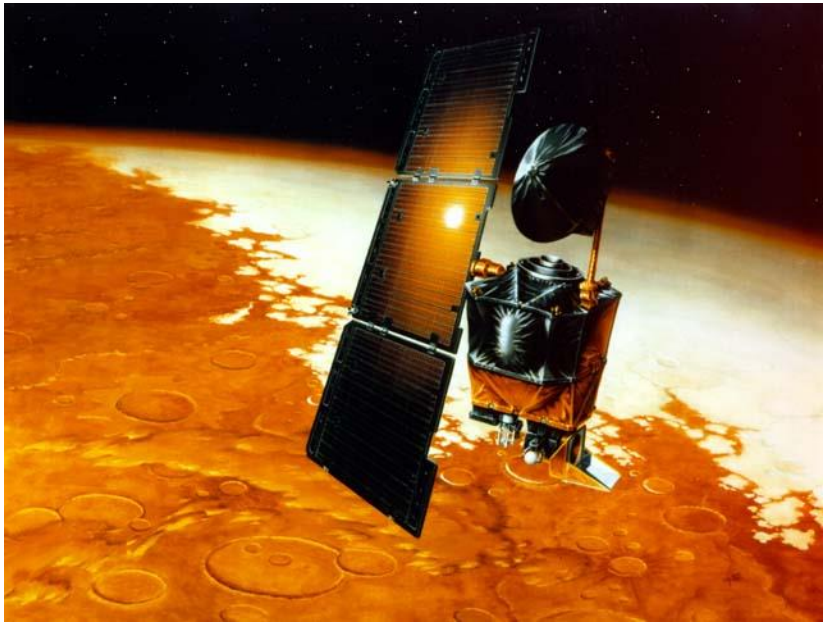
- Plošina Sleipner pro těžbu zemního plynu, která byla umístěna v Severním moři poblíž Norska, se 23. 8. 1993 zřítila kvůli chybě nepřesnému použití metody konečných prvků (**chyba metody, chyba aproximace**)
- Celková škoda cca 700 miliónů USD

Ariane 5

- Zničen 4. 6. 1996 30 sekund po startu **kvůli špatnému zápisu čísla v počítači**
- Celková škoda cca 500 miliónů USD

Mars Climate Orbiter

- Zničen 23. 9. 1999 při vstupu do atmosféry Marsu **kvůli chybě ve vstupních datech**
- Celková škoda cca 330 miliónů USD



Děkuji za pozornost!

Soutěžní otázka do soutěže ŠKOMAM Cup

- Převeďte číslo z desítkové soustavy do dvojkové: $0,\overline{1}_{10} = ?_2$
- Převeďte číslo z dvojkové soustavy do desítkové: $0,\overline{1}_2 = ?_{10}$